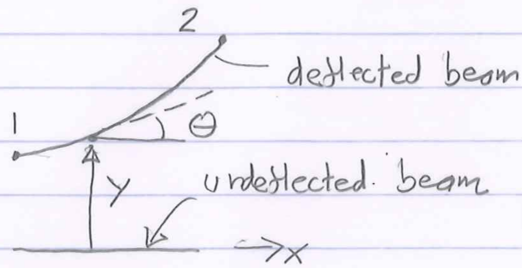
32. Moment-area method for computing beam deflections

Same assumptions as previous section, except this formulation allows EI to vary with x. Three formulas will be derived.

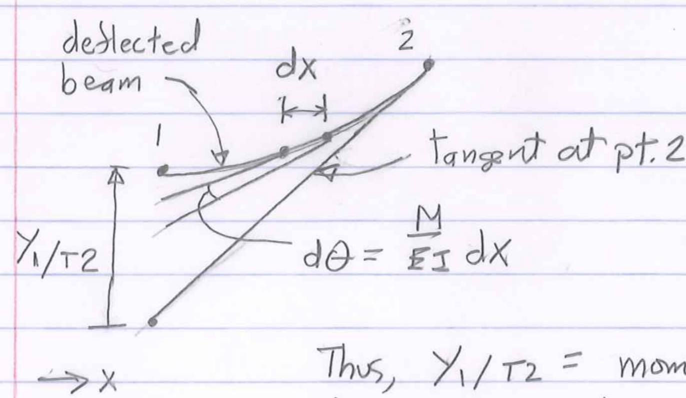


$$\theta = \frac{dy}{dx}$$

$$\frac{d\theta}{dx} = \frac{d^2y}{dx^2} = \frac{M}{EI}$$

$$\int_{pt.1}^{pt.2} d\theta = \int_{pt.1}^{pt.2} \frac{M}{EI}\,dx$$

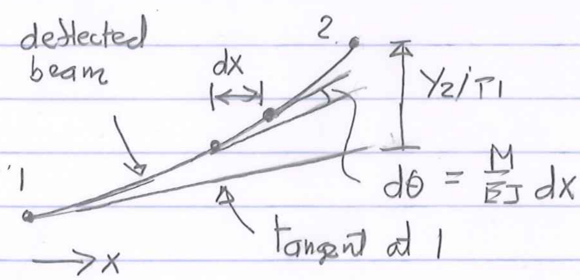
Thus, $\theta_1 - \theta_2$ = area under M/EI diagram between x_1 and x_2.



$y_{1/T2}$ = def. of pt. 1 relative to tangent at pt. 2

$$= \int_{pt.1}^{pt.2} (x - x_1)\frac{M}{EI}\,dx$$

Thus, $y_{1/T2}$ = moment of area under $\frac{M}{EI}$ diagram between x_1 and x_2 about x_1.



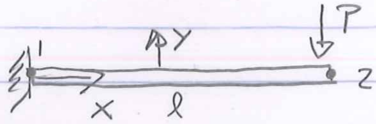
$y_{2/T1}$ = def. of pt. 2 relative to tangent at pt. 1

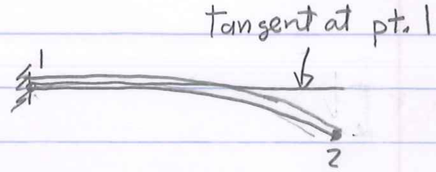
$$= \int_{pt.1}^{pt.2} (x_2 - x)\frac{M}{EI}\,dx$$

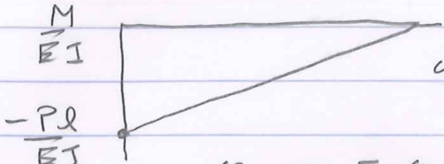
Thus, $y_{2/T1}$ = moment of area under $\frac{M}{EI}$ diagram between x_1 and x_2 about x_2.

Note that the lever arms in the last two formulas are always positive; whereas, M/EI can either be positive (as shown) or negative. $y_{1/T2}$ or $y_{2/T1}$ is positive upward.

Example:





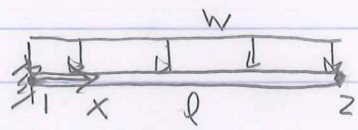


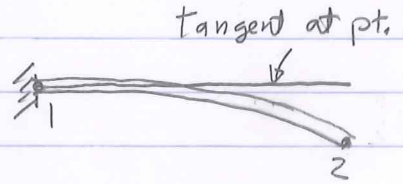
constant EI

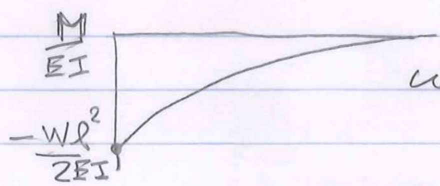
$$\theta_2 - \theta_1 = \theta_2 = -\frac{Pl}{EI}\cdot\frac{1}{2}\cdot l = -\frac{Pl^2}{2EI}$$

$$Y_2 = Y_{2/T1} = -\frac{Pl}{EI}\cdot\frac{1}{2}\cdot l\cdot\frac{2}{3}l = -\frac{Pl^3}{3EI}$$

Example:



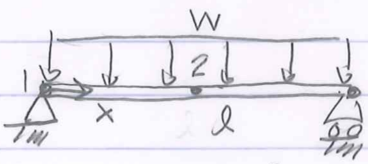


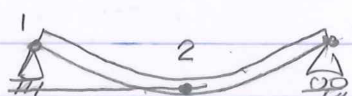


constant EI, W

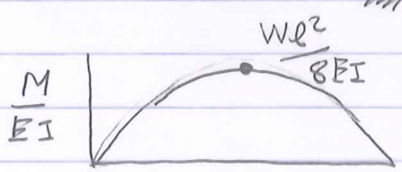
$$Y_2 = Y_{2/T1} = -\frac{Wl^2}{2EI}\cdot\frac{1}{3}\cdot l\cdot\frac{3}{4}l = -\frac{Wl^4}{8EI}$$

Example:





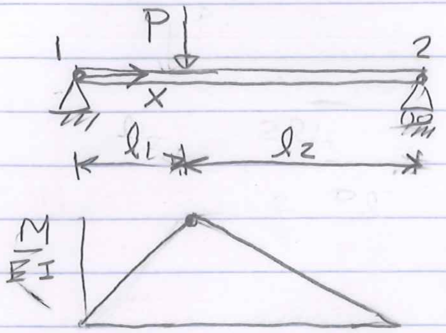
tangent at pt. 2
(2 is at mid-span)

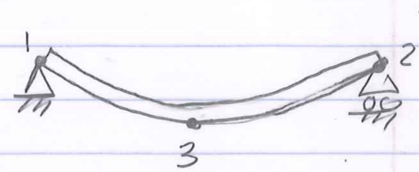


constant EI, W

$$Y_2 = -Y_{1/T2} = -\frac{Wl^2}{8}\cdot\frac{2}{3}\cdot\frac{l}{2}\cdot\frac{5}{8}\frac{l}{2} = -\frac{5}{384}\frac{Wl^4}{EI}$$

Example: Find the maximum deflection.





Point 3 is where maximum deflection occurs (not where P is applied unless $l_1 = l_2$).

Step 1: $y_{2/T1} = \int_{pt.1}^{pt.2} (x_2 - x) \frac{M}{EI}\, dx$

Step 2: $\theta_1 = -y_{2/T1} / (l_1 + l_2)$

Step 3: $\theta_3 - \theta_1 = \int_{pt.1}^{pt.3} \frac{M}{EI}\, dx$, which can be solved for x_3 since θ_1 is known and $\theta_3 = 0$

Step 4: $y_{3/T1} = \int_{pt.1}^{pt.3} (x_3 - x) \frac{M}{EI}\, dx$

Step 5: $y_3 = \theta_1 \cdot x_3 + y_{3/T1}$